

# Nonstationary seasonal model for daily mean temperature distribution bridging bulk and tails



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## Extreme Value Analysis

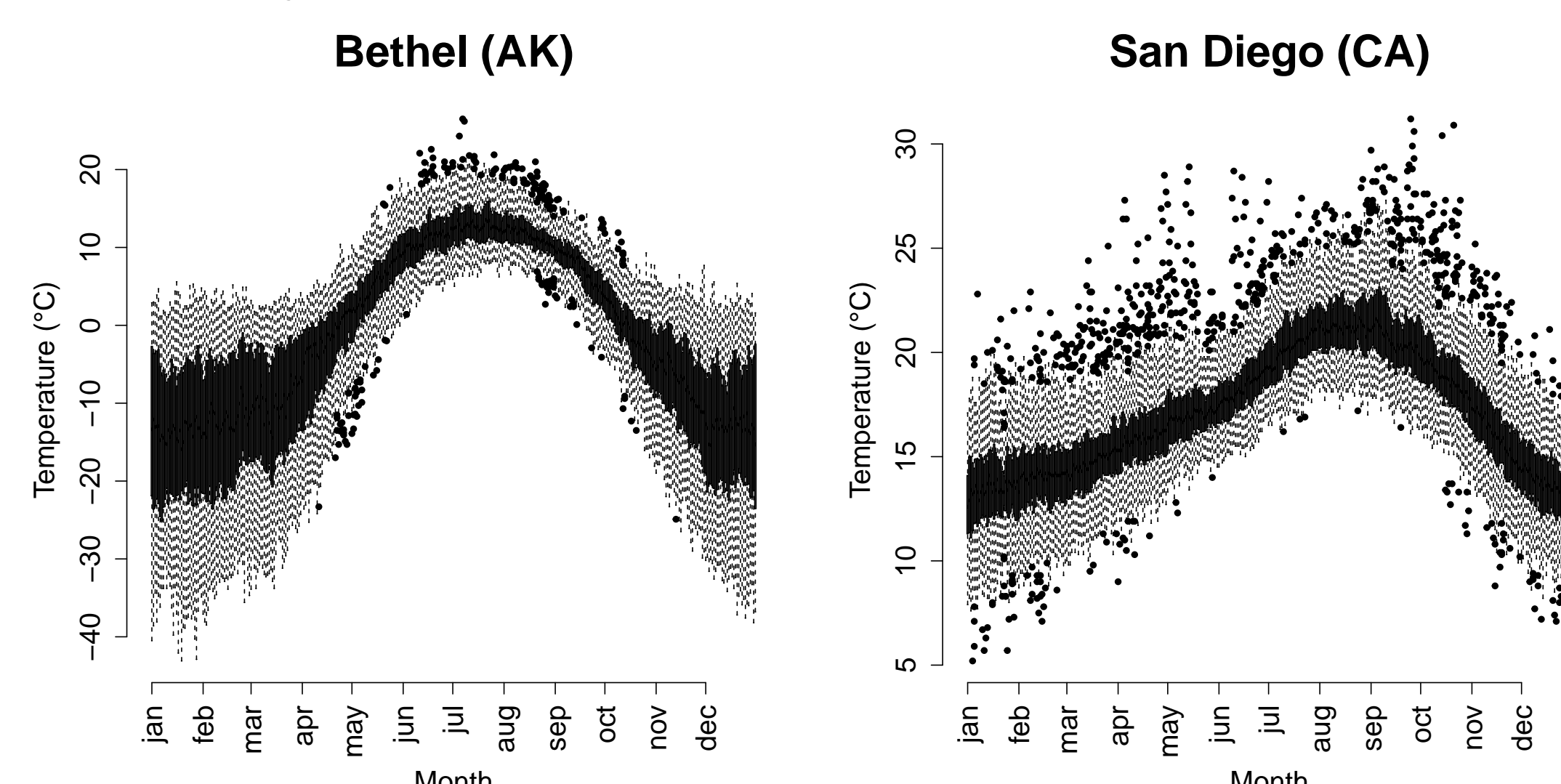
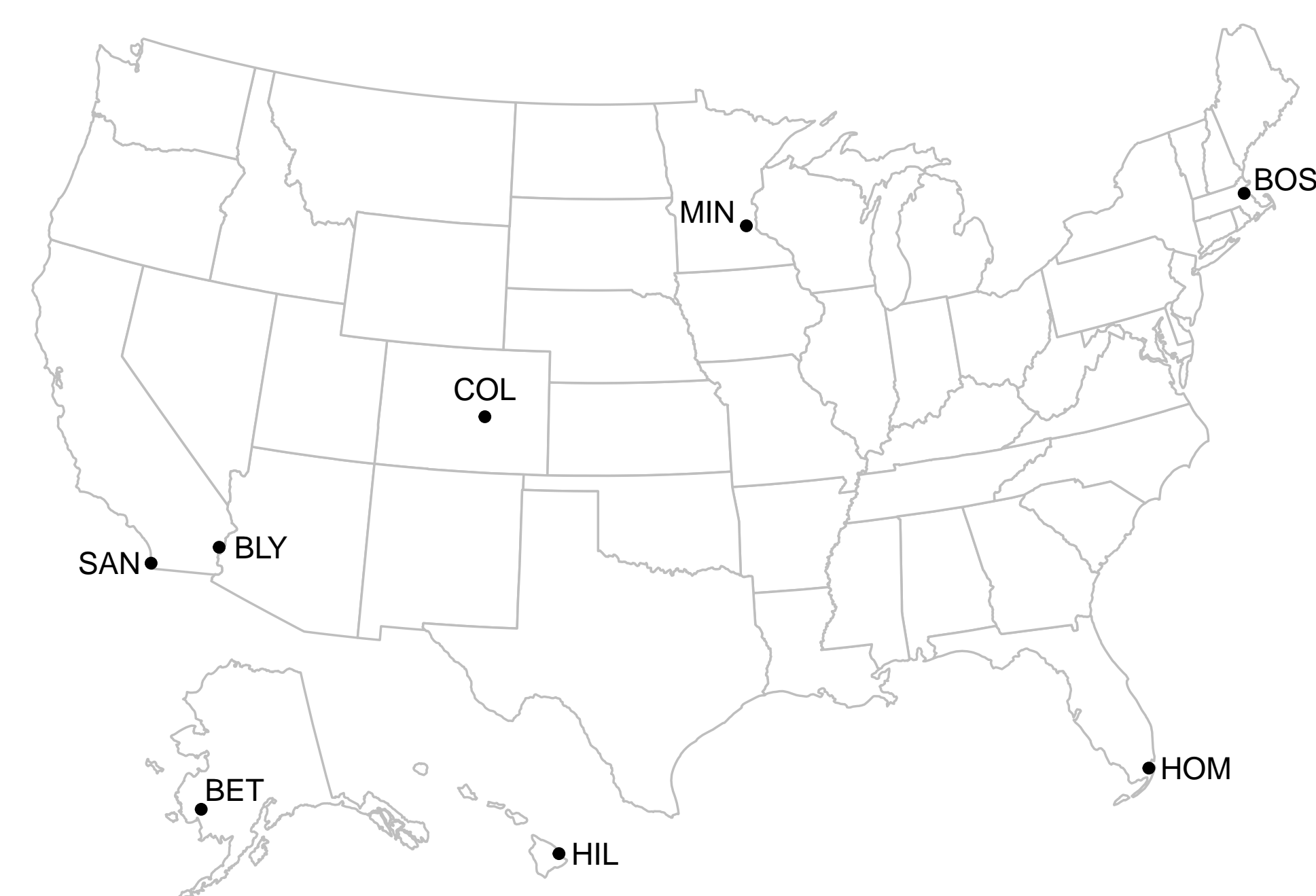
- Typical approaches are peaks-over-threshold (fit with GPD) or block-maxima (fit with GEV).
- Only consider a single tail at a time and ignore most of the available data.

## Goals

- Both tails of temperature distribution are of interest, especially due to climate change.
- Different nonstationarities (e.g., seasonality and climate change) in the bulk and tails.
- Want a single model for bulk and tails.

## Data

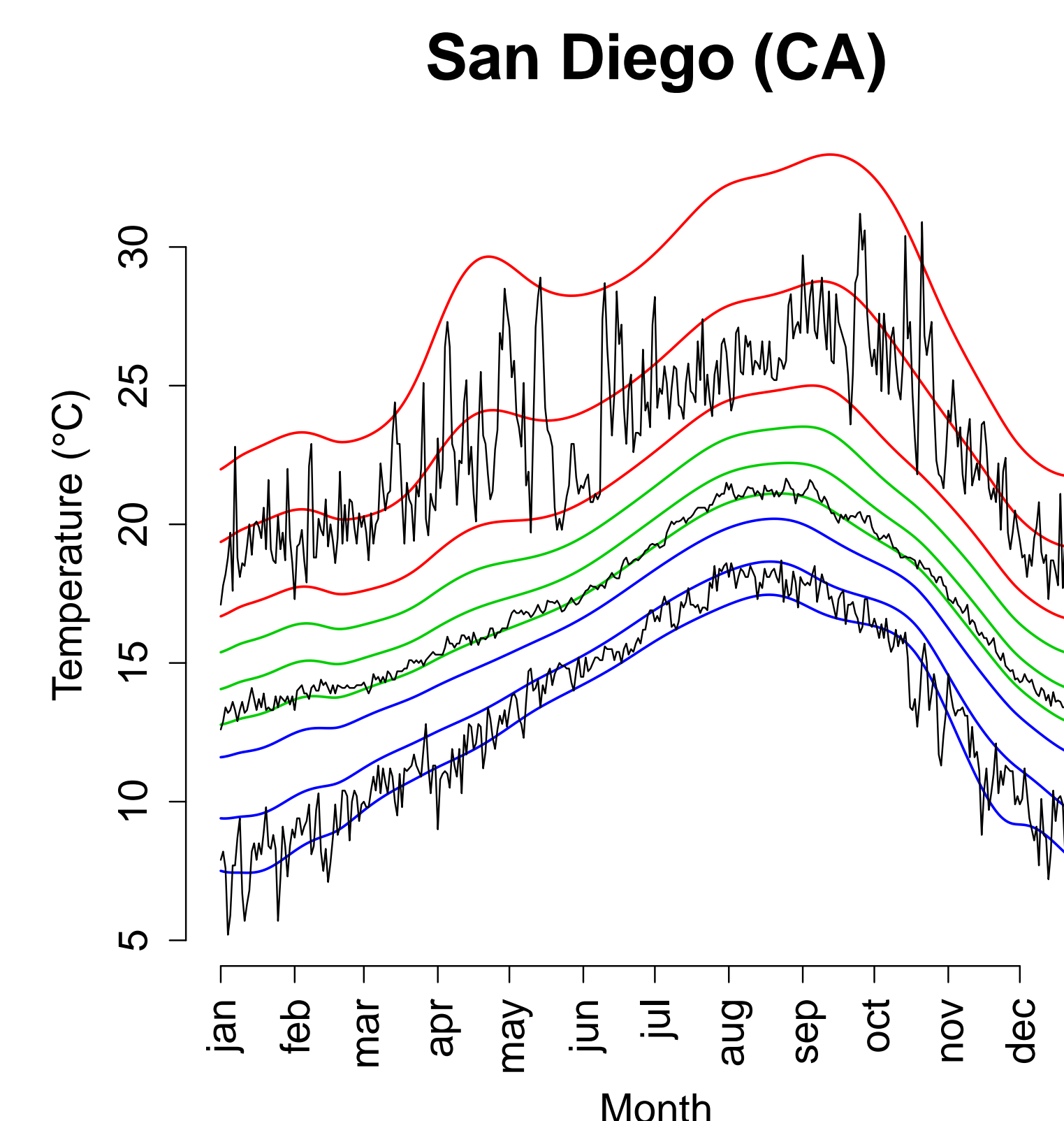
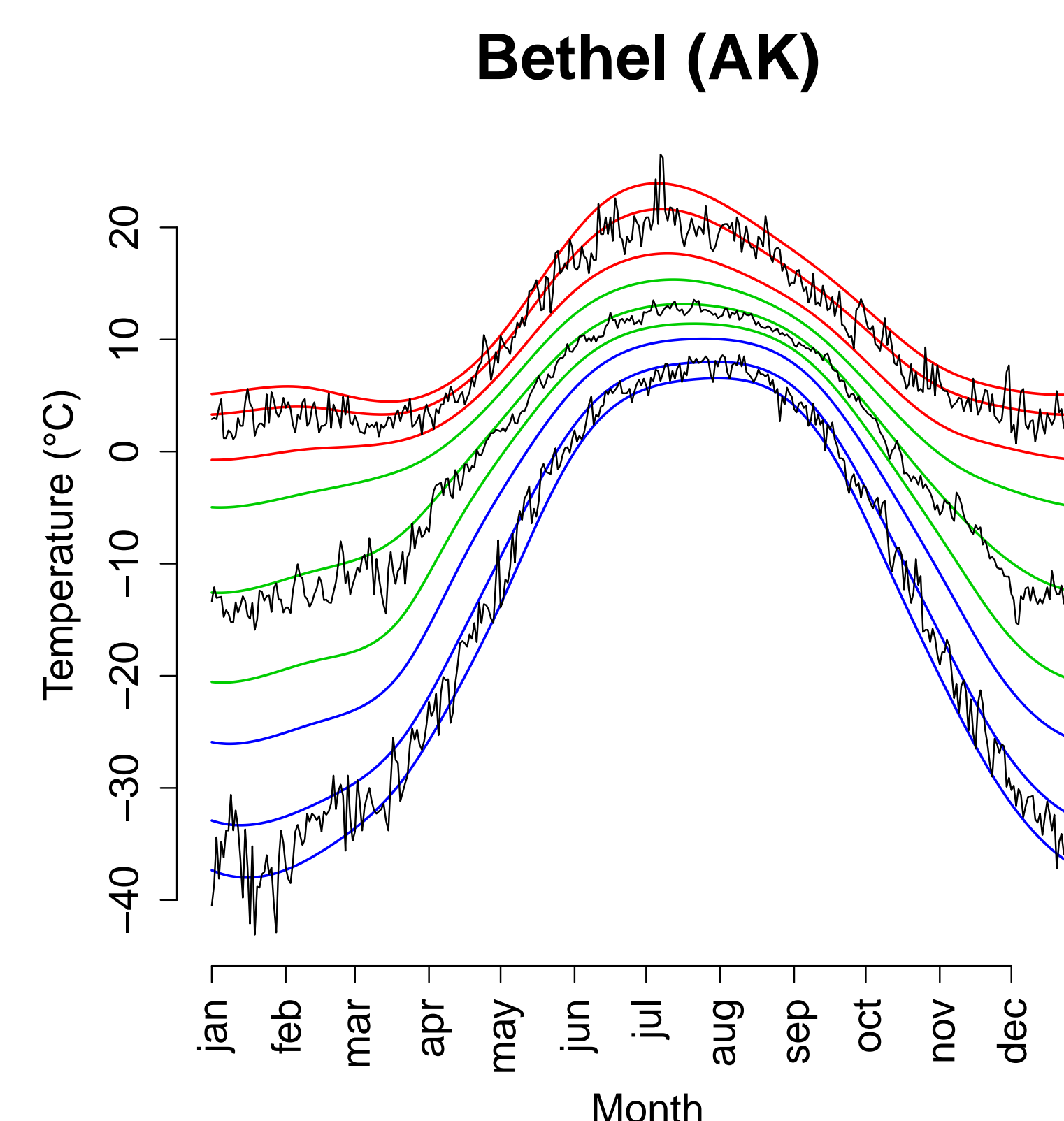
- Daily mean surface air temperature (SAT) from NCEI's Global Surface Summary of the Day.
- Eight locations with very different climates.
- Records range from early 1940's to 2020.



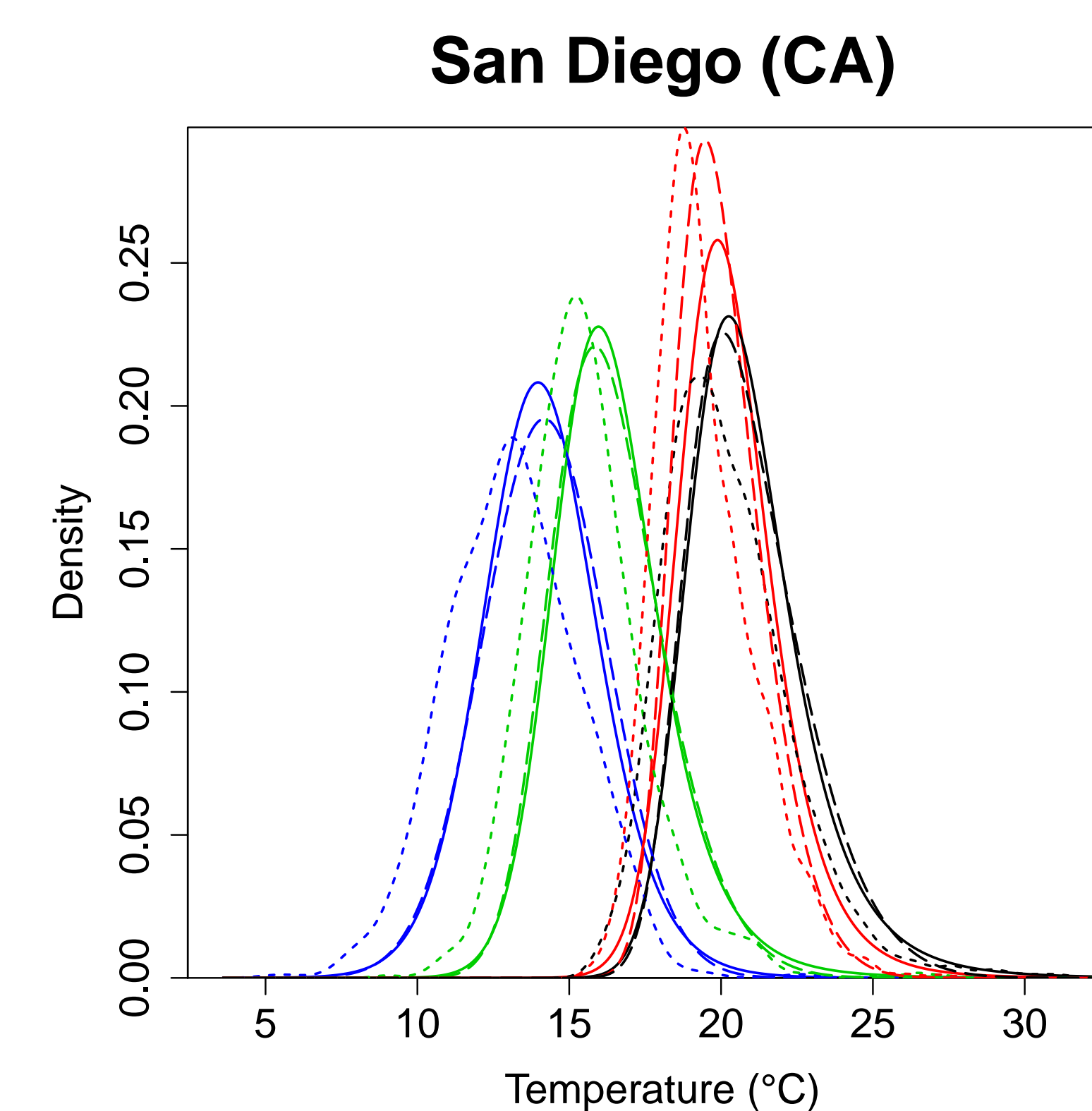
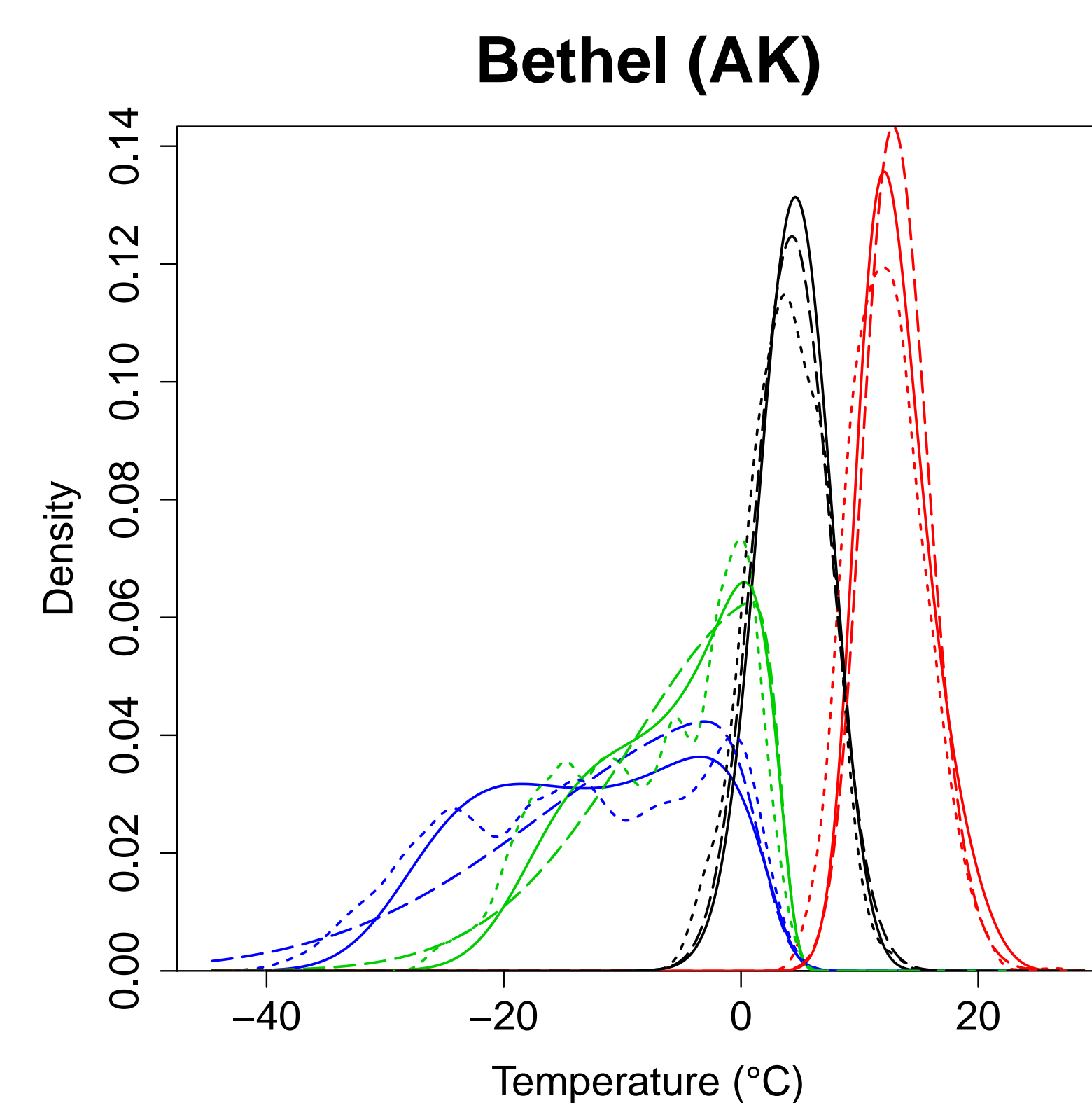
Boxplot of daily mean SAT across day of year.

## Model and Results

- Seven parameter Bulk-And-Tails (BATs) distribution (Stein 2020) has location, scale, and shape parameters for the upper and lower tails.  
*Stein, M. L. (2020). A parametric model for distributions with flexible behavior in both tails. Environmetrics.*
- Scale and location (in both tails) depend upon time-varying covariates:  
**Location:**  $\phi(d, y) = \text{seasonal}(d) + \text{trend}(y) + \text{seasonal}(d) \times \text{trend}(y)$   
**Scale:**  $\log(\tau(d)) = \text{seasonal}(d)$
- Logarithm of CO<sub>2</sub> equivalent from PRIMAP accounts for climate change.

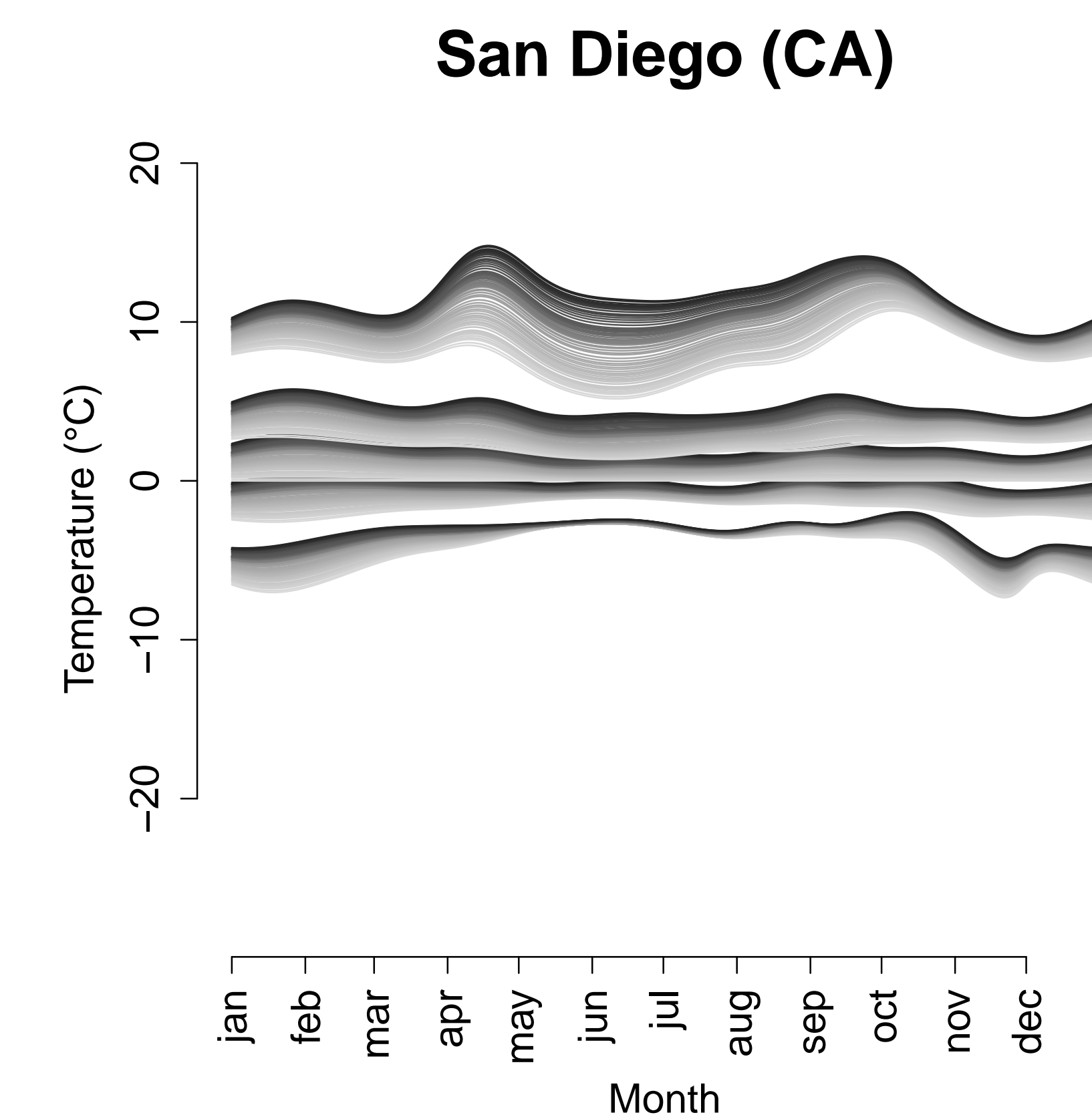
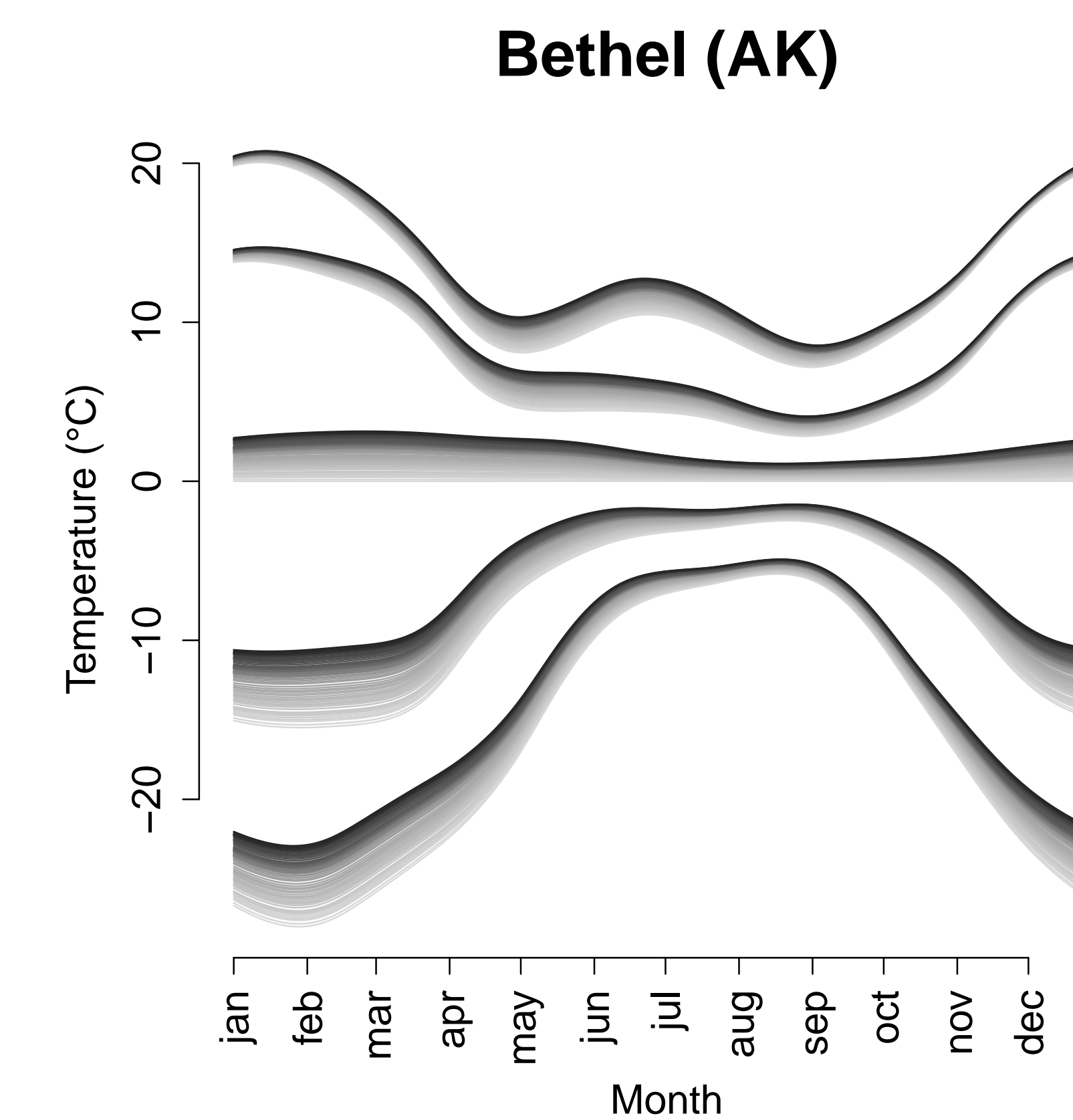


BATs quantile for the year 2020: 0.001, 0.01, 0.1 (blue), 0.25, 0.5, 0.75 (green), and 0.9, 0.99, 0.999 (red). Black lines show minimum/median/maximum observation for that day of year, taken over all years.

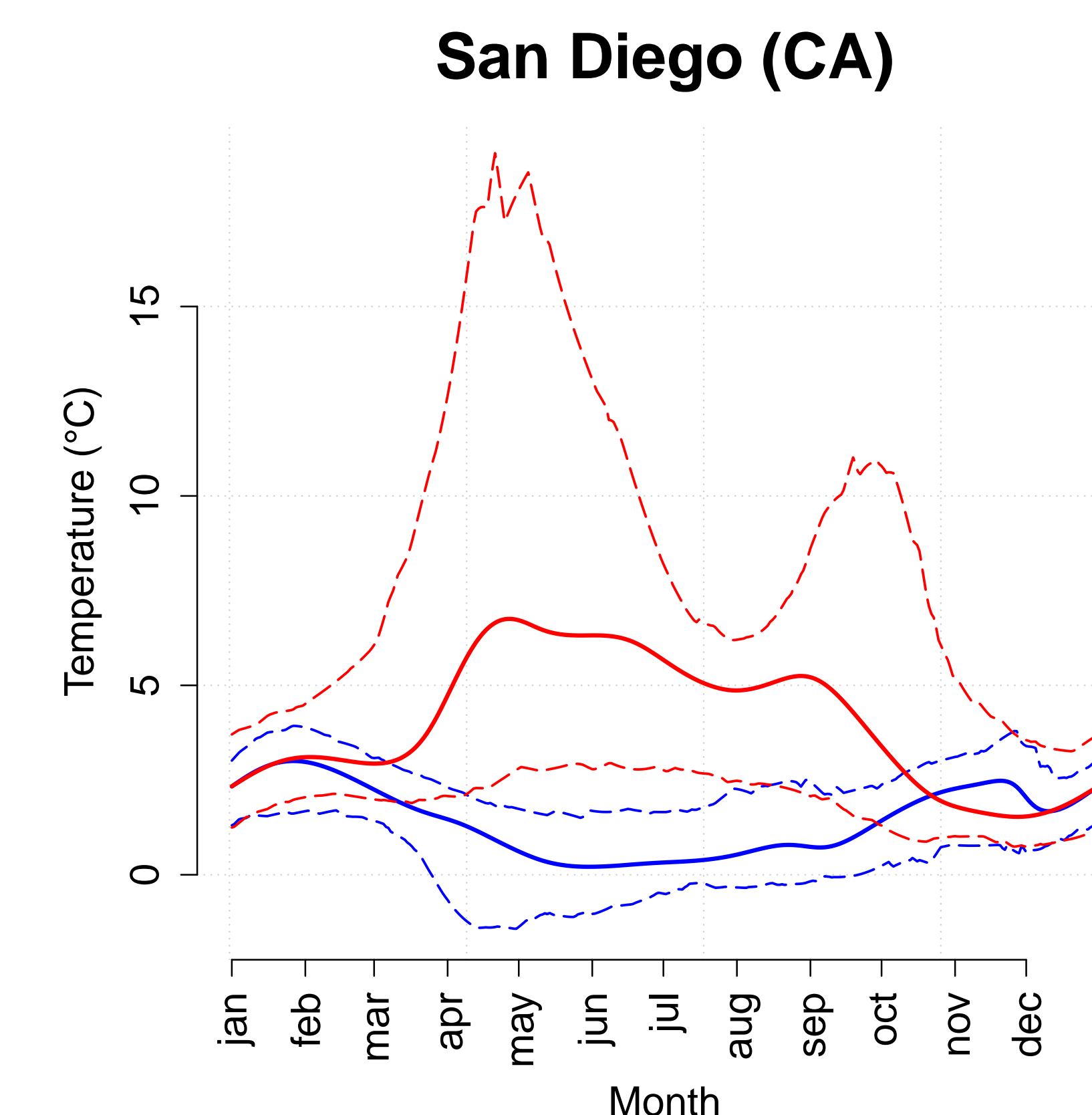
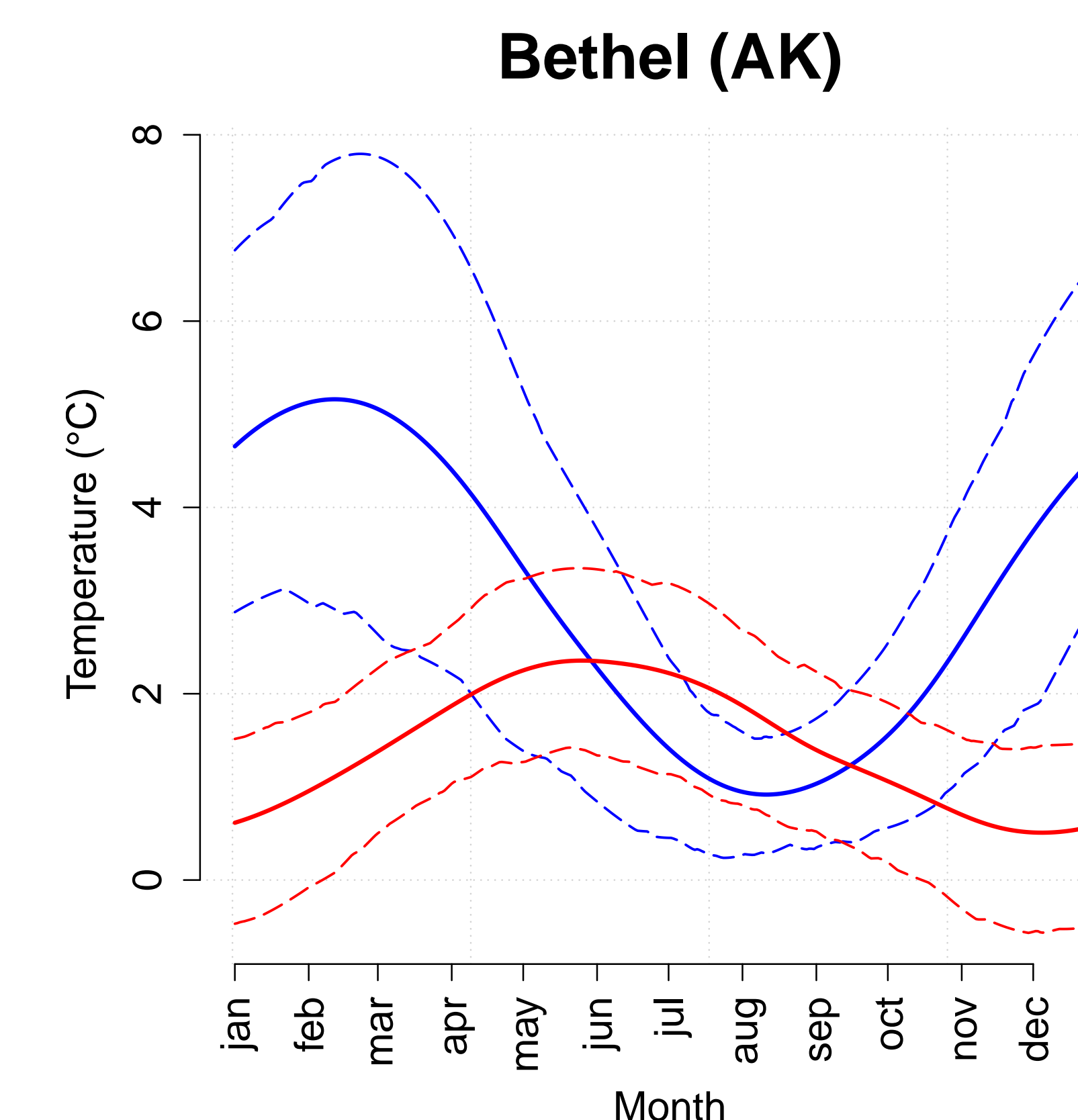


Densities for the BATs model (solid), skew-normal model (long dash), and a kernel density estimate (short dash). Blue, green, red, and black curves correspond to the first day of January, April, July, and October.

## Change in Quantiles



BATs quantile changes from first year (light) to 2020 (dark). Curves are the 0.001, 0.1, 0.5, 0.9, and 0.999 quantiles minus the 0.5 quantile at the first year.



Estimates for change in 0.001 (blue) and 0.999 (red) BATs quantiles from first year to 2020. Dashes show 95% confidence interval from bootstrapping.

## Conclusion and Future Directions

- Stein (2020) provides a flexible model for the bulk and tails of a distribution.
- Capture nonstationarity (seasonality and climate change) of daily SAT by allowing BATs parameters to depend upon time-varying covariates.
  - ▶ Could let other parameters depend on log CO<sub>2</sub> equivalent.
- Explicitly modeling time dependence would help with clustering of extremes.
- Possible stepping-stone for space-time extremes model.